B.Sc-part-I (H) Set Theorem 13-07-2020
Paper-I (H) General form of De Morgan's Laws.

Q.1. De Morgan's laws in general form prove that
(I) $\left(\bigcup_i A_i\right)' = \bigcap_i A_i'$ (II) $\left(\bigcap_i A_i\right)' = \bigcup_i A_i'$

Proof: I. Let $x \in \left(\bigcup_i A_i\right)' \Rightarrow x \notin \left(\bigcup_i A_i\right)$

$\Rightarrow x \notin (A_1 \cup A_2 \cup A_3 \cup \cdots\cdots)$
$\Rightarrow x \notin A_1 \wedge x \notin A_2 \wedge x \notin A_3 \cdots\cdots$
$\Rightarrow x \in A_1' \wedge x \in A_2' \wedge x \in A_3' \cdots\cdots$
$\Rightarrow x \in (A_1' \cap A_2' \cap A_3' \cap \cdots\cdots)$
$\Rightarrow x \in \left(\bigcap_i A_i'\right)$

$\therefore \left(\bigcup_i A_i\right)' \subseteq \bigcap_i A_i'$ ——— (1)

Again, let $y \in \bigcap A_i' \Rightarrow y \in (A_1' \cap A_2' \cap A_3' \cap \cdots\cdots)$

$\Rightarrow y \in A_1' \wedge y \in A_2' \wedge y \in A_3' \cdots\cdots$
$\Rightarrow y \notin A_1 \wedge y \notin A_2 \wedge y \notin A_3 \cdots\cdots$
$\Rightarrow y \notin (A_1 \cup A_2 \cup A_3 \cup \cdots\cdots)$
$\Rightarrow y \notin \bigcup_i A_i$
$\Rightarrow y \in \left(\bigcup_i A_i\right)'$

$\therefore \bigcap_i A_i' \subseteq \left(\bigcup_i A_i\right)'$ ——— (2)

Now from (1) and (2), by equality of sets.
$\left(\bigcup_i A_i\right)' = \bigcap_i A_i'$

Proved.

B.Sc. part I (H) Set Theory 13-07-2022
Paper-I De Morgan's Laws in general form.

Q.1 (ii) Let $x \in (\bigcap_i A_i)' \Rightarrow x \notin (\bigcap_i A)$

$\Rightarrow x \notin (A_1 \cap A_2 \cap A_3 \cap \cdots)$

$\Rightarrow x \notin A_1 \vee x \notin A_2 \vee x \notin A_3 \cdots$

$\Rightarrow x \in A_1' \vee x \in A_2' \vee x \in A_3' \cdots$

$\Rightarrow x \in (A_1' \cup A_2' \cup A_3' \cup \cdots)$

$\Rightarrow x \in (\bigcup_i A_i')$

$\therefore (\bigcap_i A_i)' \subseteq \bigcup_i A_i'$ —— (1)

Again, let $y \in \bigcup A_i' \Rightarrow y \in (A_1' \cup A_2' \cup A_3' \cup \cdots)$

$\Rightarrow y \in A_1' \vee y \in A_2' \vee y \in A_3' \cdots$

$\Rightarrow y \notin A_1 \vee y \notin A_2 \vee y \notin A_3 \cdots$

$\Rightarrow y \notin (A_1 \cap A_2 \cap A_3 \cap \cdots)$

$\Rightarrow y \notin (\bigcap_i A_i)$

$\Rightarrow y \in (\bigcap_i A_i)'$

$\therefore \bigcup_i A_i' \subseteq (\bigcap_i A_i)'$ —— (2)

Now, from (1) and (2), by equality of sets.

$(\bigcap_i A_i)' = \bigcup_i A_i'$

Proved.